Tutorial 6:

Preliminary:

Outstanding Balance: Set $OB_0 = L = k_1 v + k_2 v^2 + \cdots + k_n v^n$

after first payment k_1, $OB_1 = OB_0(1+i) - k_1$,

after second payment k_2, $OB_2 = OB_1(1+i) - k_2 = (OB_0(1+i) - k_1)(1+i) - k_2$

$= OB_0(1+i)^2 - k_1(1+i) - k_2$

⋮

after t^{th} payment k_t, $OB_t = OB_{t-1}(1+i) - k_t$

$= OB_0(1+i)^t - k_1(1+i)^{t-1} - \cdots - k_{t-1}(1+i) - k_t$

if $k_1 = k_2 = \cdots = k_t = k$, (retrospective) $= OB_0(1+i)^t - k\, s_{\overline{t|}\,i} = k\, a_{\overline{n-t|}\,i}$ (prospective)

Interest paid: $I_t = OB_{t-1} \times i$

Principal repaid: $PR_t = OB_{t-1} - OB_t = OB_{t-1} - OB_{t-1}(1+i) + k_t = k_t - OB_{t-1} \cdot i = k_t - I_t$.

if $k_1 = k_2 = \cdots = k_n = k$, $I_t = k\, a_{\overline{n-t+1|}\,i} \times i = k(1 - v^{n-t+1})$

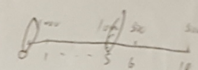
$PR_t = k - I_t = k v^{n-t+1}$ $\Rightarrow$ $\frac{PR_t}{PR_n} = \frac{k v^{n-t+1}}{k v^{n-n+1}} = v^{n-t}$
$= (1+i)^{t-n}$

Exercises:

3.1.1.

$k_1 = k_2 = \cdots = k_5 = 1000$, $k_6 = k_7 = \cdots = k_{10} = 500$, $i = 10\%$ "start a new from now".

(i) $L = 1000\, a_{\overline{5|}10\%} + 500\, a_{\overline{5|}10\%} \cdot v^5 = 4{,}967.68$



(ii) $OB_3 = OB_0(1+i)^3 - 1000\, s_{\overline{3|}\,i} = 4967.68 \times (1.1)^3 - 1000\, s_{\overline{3|}10\%} = 3301.98$.

(iii) $I_4 = OB_3 \times i = 3301.98 \times 0.1 = 330.20$.

$PR_4 = k_4 - I_4 = 1000 - 330.20 = 669.80$.

(iv) $OB_8 = 500\, a_{\overline{2|}10\%} = 867.77$.

3.1.4.

(i) $L = 20{,}000$, $n = 4 \times 12 = 48$, $i^{(12)} = 6\%$, $i = 0.5\%$.

assume monthly payment K.

$L = K a_{\overline{12|}0} + K v_0^{12} a_{\overline{36|}0.5\%} = 44.87K \Rightarrow K = 445.72$.

$OB_{12} = K a_{\overline{36|}0.5\%} = 14{,}651$. $L(1+i)^{12} - K s_{\overline{12|}\,i}$

(ii) $j_1^{(12)} = 3\%$, $j_1 = 0.25\%$, $j_2^{(12)} = 5\%$, $j_2 = 0.42\%$, assume payment P.

$P a_{\overline{12|}0.25\%} + P v_{0.25\%}^{12} a_{\overline{36|}0.42\%} = L \Rightarrow P = 452.61$.

$OB_{12} = P a_{\overline{36|}0.42\%} = 15{,}102$.

3.2-5.

(a) $n = 5 \times 2 = 10$, "immediate". $K = 200$,

$PR_1 = 156.24 = 200\, v_j^{\,n-t+1} = 200\, v_j^{\,10} \Rightarrow v_j^{\,10} = 0.7812 \Rightarrow j = 0.025$, semiannual rate.

monthly rate $(1+i)^{12} = (1+j)^2 \Rightarrow i = (1+j)^{\frac{1}{6}} - 1 \Rightarrow i^{(12)} = 12 \times i = 0.0495$.

(b) $n = 48$, $K = 200$. $I_1 + I_2 + \cdots + I_{12} = 983.16$, $PR_{37} + PR_{38} + \cdots + PR_{48} = 2215.86$.

$PR_1 + PR_2 + \cdots + PR_{12} = (K - I_1) + (K - I_2) + \cdots + (K - I_{12}) = 2400 - 983.16 = 1416.84$.

$$\frac{PR_{37} + \cdots + PR_{48}}{PR_1 + \cdots + PR_{12}} = (1+i)^{36} = \frac{2215.86}{1416.84} \Rightarrow i = 0.0125 \Rightarrow i^{(12)} = 0.15.$$

$\underline{OB_t} = [K_1 v + K_2 v^2 + \cdots + K_t v^t + K_{t+1} v^{t+1} + \cdots + K_n v^n](1+i)^t$

$- K_1 (1+i)^{t-1} - \cdots - K_{t-1}(1+i) - K_t$

$= K_{t+1} v + K_{t+2} v^2 + \cdots + K_n v^{n-t}$ prospective